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ADDENDUM

Creeping flow in two-dimensional networks

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Two extensions have come to light since the appearance of this paper, on fluid flow in two-dimensional random networks of the 'ball-and-stick' variety.

First, we pointed out in the paper that most results could be taken over directly in the 3-dimensional case, the only missing ingredient being an expression for the pressure drop across a spherical pore. In the 2-dimensional case, our earlier result (24*c*) for circular pores may be shown to be equivalent to the pressure drop for a straight channel emptying into a half-plane. For 3 dimensions it then suffices to know the pressure drop for a cylindrical channel emptying into a half-space. This latter problem has been thoroughly investigated by Z. Dagan, S. Weinbaum & R. Pfeffer (*J. Fluid Mech.* **115** (1982), 505) who in effect find, as a 3-dimensional replacement for (24*c*),

$$p_j = (2\mu l/3r_j^4)\, Q_j.$$

The networks discussed in the paper were not fully random, in that the pore centres were constrained to lie on the nodes of a regular lattice, but we may generalize our results as follows. In place of the case-by-case calculations of §5 for various specific lattices, consider, as in figure 1, an arbitrary lattice in which the nodes are randomly distributed in space and connected together in a random fashion. We require the permeability of this lattice, under the assumption that the flux in bond b is related to the pressure drop Δp_b across its ends by $Q_b = g_m \Delta p_b$. We suppose the two ends of the network are held at fixed pressures, and define an average pressure gradient $\langle \nabla p \rangle$ as the difference in end pressures divided by sample length. Choose a plane P perpendicular to $\langle \nabla p \rangle$, and compute the total fluid flux Q_T passing through the plane. The latter is just the sum of the individual bond fluxes Q_b for bonds b crossing P. The obvious first approximation, suggested to the author by B. I. Halperin, is obtained by assuming that the local pressure field is the same as the average pressure field, in which case Δp_b is just the projection of the bond length $\boldsymbol{l}_b$ in the direction of $\langle \nabla p \rangle$. Thus

$$Q_T(P) = g_m \sum_{b \cap P} \boldsymbol{l}_b \cdot \langle \nabla p \rangle.$$

The average fluid velocity is computed by dividing $Q_T(P)$ by the area $A(P)$ of the sample plane, and averaging over different planes P. The permeability is the proportionality constant relating $\langle \nabla p \rangle$ to the average velocity and so

$$k = g_m \left\langle \frac{1}{A(P)} \sum_{b \cap P} \boldsymbol{l}_b \cdot \hat{\boldsymbol{n}} \right\rangle,$$

where $\hat{\boldsymbol{n}}$ is a unit vector orthogonal to P. If the network is statistically homogeneous and isotropic, k will be independent of the choice of $\hat{\boldsymbol{n}}$. For a 2-dimensional network, t is given by the same construction with A replaced by the sample width. Note that k always has the dimensions of area. The approximation of equating local and average

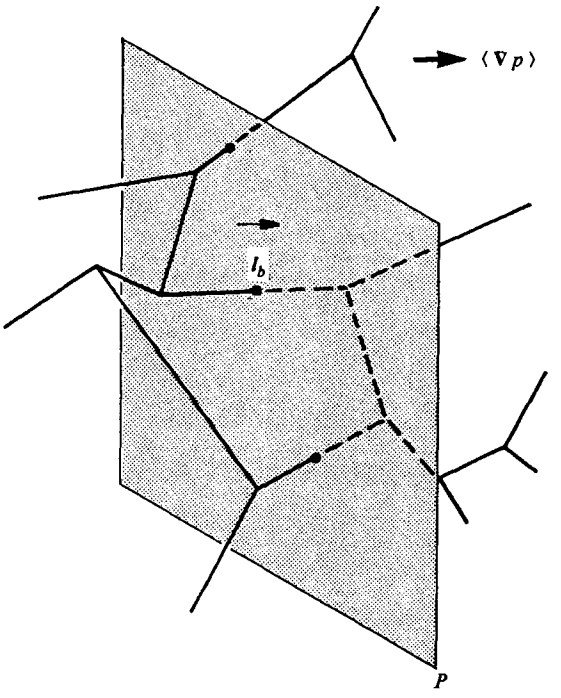

FIGURE 1

pressure fields (which we have also used to obtain g_m) should be correct to first order in the pressure fluctuations. When these fluctuations are absent, as in a *regular* lattice, this replacement should be exact, and one may check that (4) agrees with the results for t stated in the paper (see tables 4 and 5) for various 2- and 3-dimensional lattices.